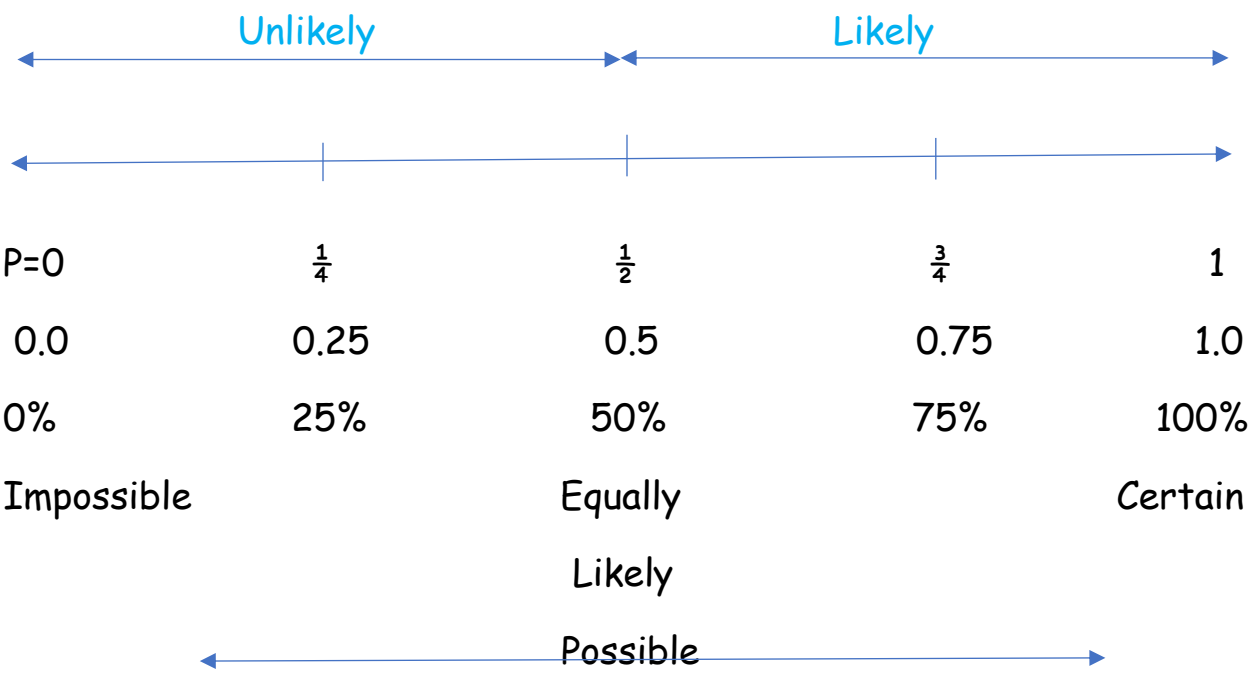
Hello again Grade 6!

For this week, we will move on to a new topic. Keep in mind that the work here is for the week. Space it out, and just try your best. Now that we have developed an understanding of fractions, decimals and percents and how they relate to each other, lets take a look at some Probabilities.

A little background from Grade 5 reminds us that Probabilities are expressed in terms of how likely an event will occur. We used words like those shown below, to describe the likelihood of an event occurring.



In the above diagram, if the Probability of an event is zero (0), that means it is impossible that the event will ever occur. For example, pigs flying! It's impossible, so it will NEVER happen 😊

An event with a Probability of $\frac{1}{2}$ is called an equally likely event, meaning each outcome will have the same chance of occurring. For

example, if you toss a coin, the probability of getting a head is 1 out of the two possibilities (heads or tails). The probability of getting a tail is also 1 out of two possibilities, so both probabilities are $\frac{1}{2}$.

$P_T(\text{heads}) = \frac{1}{2}$ $P_T(\text{tails}) = \frac{1}{2}$

An event with a Probability of 1, means the event will always occur EVERY time, so it is called a certain event. For example, if you have a bag of blue marbles, the probabitily of choosing a blue marble from the bag is 1, or 100%.

$P_T(\text{blue}) = 1$

All Probabilities between 0 and 1 are called POSSIBLE events.

**Note from the diagram on the first page, Probabilities can be expressed as fractions, decimals and percents.

Also, when talking about Probabilities, when referring to outcomes, we are referring to the possible results of an event. For example, a coin

has two possible outcomes – heads and tails. A six sided die has six possible outcomes – 1,2,3,4,5,6.

Now that we have done some review and reminding from last year, lets look at some new ideas. 😊

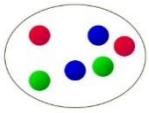
By definition, Probability is the measure of how likely an event is to occur. It is about predictions of events over the long term. We will look at two kinds of Probabilities here.

1. Theoretical Probability – refers to what should happen and is expressed by finding

$$P_T(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

Note in the above: the small "T" refers to Theoretical Probability and the (E) refers to the event you are considering.

For example: In the photo, there are 2 red marbles, 2 green marbles, and 2 blue marbles.



The possible outcomes here are red, green and blue.

The Theoretical Probability of choosing red is

$P_T(\text{red}) = 2/6$ *fraction can be reduced by dividing both the numerator and denominator by 2.

$= 2\div2 \,/\, 6\div2$

$= 1/3$

Included are your text pages. There are some examples under the connect to help you understand and review Theoretical probability and some exercises to try.

Connect

Jamie and Alexis are playing *Predicting Products*.
They take turns to roll 2 dice, each labelled 1 to 6.
If the product of the 2 numbers rolled is odd, Jamie gets a point.
If the product is even, Alexis gets a point.
The first person to get 20 points wins.
Who is more likely to win?

Jamie	Alexis
Odd Product	Even Product

Here is one way to help predict the winner:

Organize the possible outcomes in a table.
Each number on a die has an equal chance of being rolled.

X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

From the table:
- There are 36 possible outcomes.
- 27 outcomes are even products.
- 9 outcomes are odd products.

We say: The **probability** of getting an even product is 27 out of 36.
We write the probability of an even product as a fraction: $\frac{27}{36}$

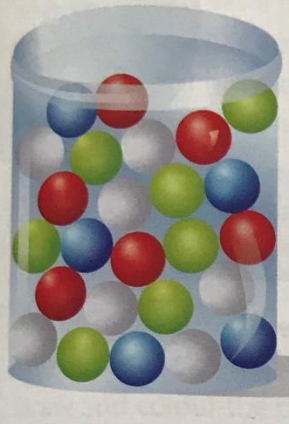
We say: The probability of getting an odd product is 9 out of 36.
We write the probability of an odd product as: $\frac{9}{36}$

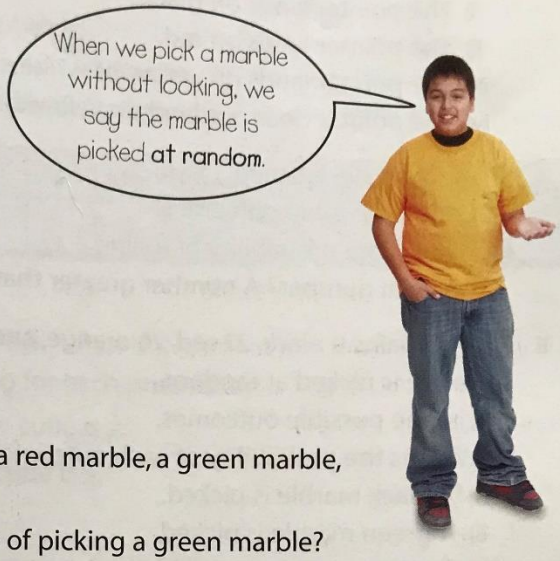
Each of these probabilities is a **theoretical probability**.
A theoretical probability is the likelihood that an outcome will happen.

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

The probability that Alexis wins is $\frac{27}{36}$.
The probability that Jamie wins is $\frac{9}{36}$.
Since $\frac{27}{36} > \frac{9}{36}$, Alexis is more likely to win.

➤ A jar contains 5 blue marbles, 6 red marbles, 7 green marbles, and 7 white marbles.
Without looking, a student picks a marble from the jar.





- What are the possible outcomes?
  The outcomes are: a blue marble, a red marble, a green marble, and a white marble.
- What is the theoretical probability of picking a green marble?
  Each marble has an equal chance of being picked.
  There are 7 green marbles, so there are 7 favourable outcomes.
  The total number of marbles is:

  $5 + 6 + 7 + 7 = 25$

  So, there are 25 possible outcomes.
  The theoretical probability of picking a green marble is $\frac{7}{25}$.

Practice

1. A paper bag contains 2 green tiles, 4 yellow tiles, and 1 blue tile. Liz draws a tile without looking.
   a) List the possible outcomes.
   b) What is the theoretical probability that the tile is:
      i) green? ii) yellow? iii) blue?

2. There are 13 girls and 17 boys in a Grade 6 class. The teacher puts each student's name into a hat, then draws one name. The student whose name is drawn will be the first to present her or his speech. What is the theoretical probability that a girl will present first?

3. Jade spins the pointer on this spinner.

a) List the possible outcomes.

b) What is the theoretical probability of each outcome?

i) The pointer lands on black.

ii) The pointer lands on red.

iii) The pointer lands on yellow or white.

iv) The pointer does not land on yellow.

4. Shen rolls a die labelled 1 to 6.

a) List the possible outcomes.

b) What is the probability of rolling a 1? An even number? A number greater than 4?

We usually say *probability* instead of theoretical probability.

5. A jar contains 9 black, 22 red, 26 orange, and 13 green marbles. A marble is picked at random.

a) List the possible outcomes.

b) What is the probability of each outcome?

i) A black marble is picked.

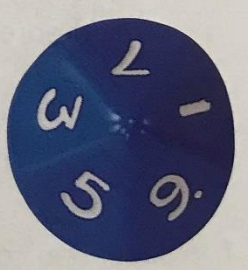
ii) A green marble is picked.

iii) A red or an orange marble is picked.

6. A letter is chosen at random from each word listed below. In each case, what is the probability that the letter chosen is a vowel?

a) Yukon b) Saskatchewan c) Nunavut d) Manitoba

7. An object with 10 congruent faces is a regular decahedron. Shannon and Joshua roll a decahedron labelled 1 to 10.



a) List the possible outcomes.

b) What is the probability Shannon rolls an odd number?

c) Joshua says there is a probability of $\frac{1}{5}$ for rolling a number with a certain digit. What is the digit?

8. At a carnival, you can choose one of these wheels to spin. To win a prize on the first wheel, the pointer must land on a star. To win a prize on the second wheel, the pointer must land on a happy face. Which wheel would you choose to spin? Use words and numbers to explain your answer.

9. This table shows the number of birthdays each month for a Grade 6 class.
A student is picked at random.
What is the probability of each event?

a) The student has a birthday in March.
b) The student has a birthday in October.
c) The student has a birthday in June, July, or August.
d) The student does not have a birthday in December.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number of Students	2	4	3	1	5	3	2	3	3	1	1	2

10. A bag contains 6 cubes.
The cubes are coloured blue and yellow.
Draw and colour the cubes in the bag for each probability:

a) The probability of picking a yellow cube is $\frac{1}{6}$.
b) The probability of picking a blue cube is $\frac{3}{6}$.

To follow up with some online activities for Theoretical Probabilities, go to www.netmath.ca

a. Vocabulary – Probability and Statistics
b. Determining the probability of an event 1

2. Experimental Probability – refers to what actually happens as a result of conducting an experiment several times and is expressed by finding

$$P_E(E) = \frac{\text{Number of times an outcome occurs}}{\text{Number of times the experiment is conducted}}$$

Note in the above: the small "E" refers to Experimental Probability and the (E) refers to the event you are considering. Again, I'll use your text to provide examples and practice.

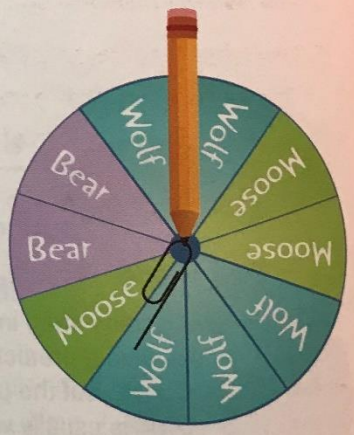
LESSON 7

Experimental Probability

A die labelled 1 to 6 is rolled.
What is the theoretical probability of rolling a 3?
How do you know?

Explore

Your teacher will give you a large copy of this spinner.
You will need an open paper clip as a pointer and a sharp pencil to keep it in place.



- Suppose the pointer is spun.
  What is the theoretical probability of the pointer landing on Wolf? Landing on Bear? Landing on Moose?
  Order these probabilities from greatest to least.
- Conduct the experiment 50 times.
  Record your results in a tally chart.
  In the last column, write the total as a fraction of 50.

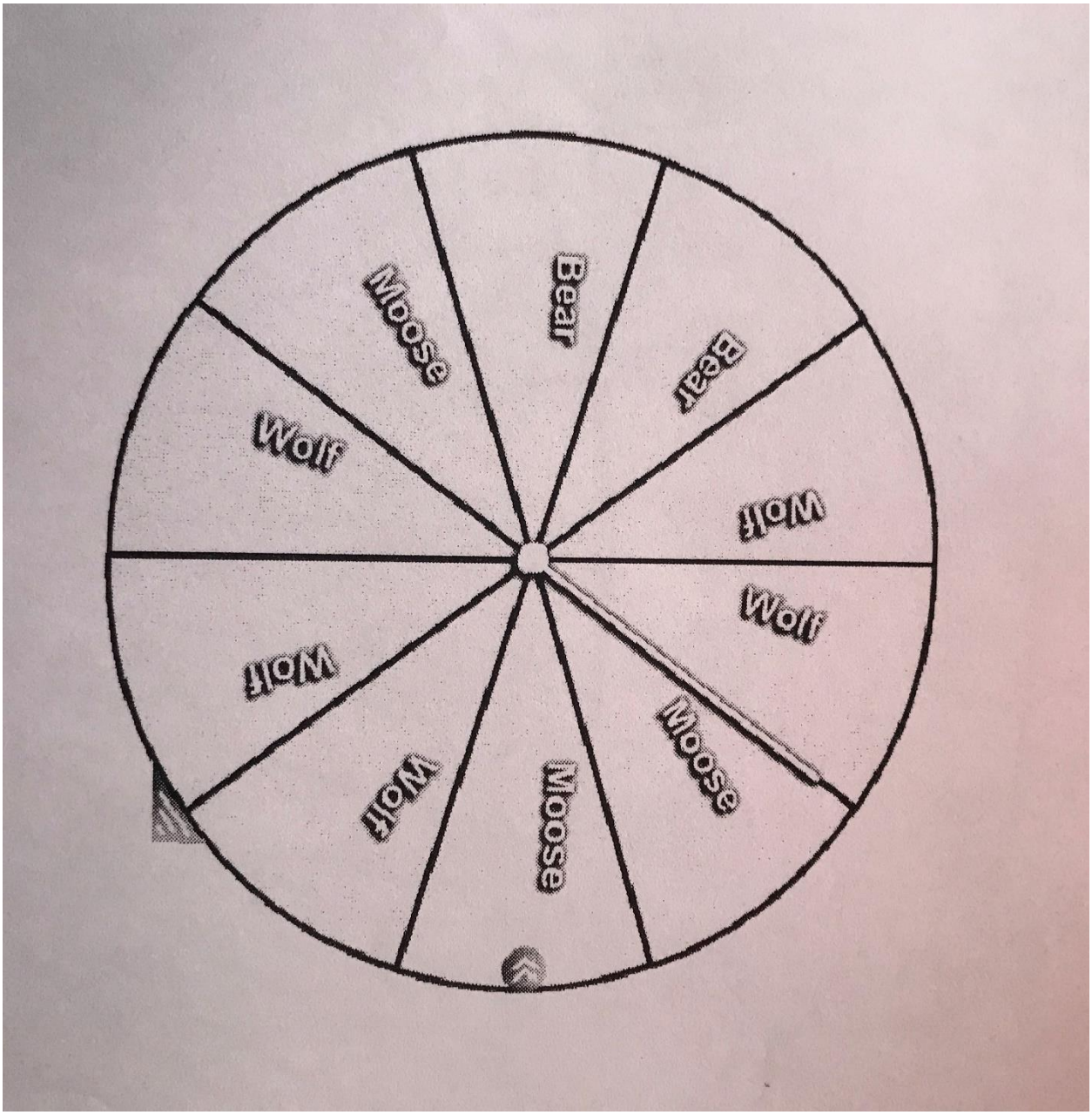
Sector	Tally	Total	$\frac{\text{Total}}{50}$
Wolf			
Bear			
Moose			

- Order the fractions from greatest to least.
  How does this order compare with the order of the theoretical probabilities?

Show and Share

Combine your results with those of another pair of students to get 100 trials.
How do the experimental results compare with the theoretical probabilities now?

If you can print and want to try the experiment:



Connect

Jenny and Morningstar put coloured cubes into a bag. They used 4 blue, 2 red, 2 green, and 2 yellow cubes. A cube is picked from the bag at random. The theoretical probability that a blue cube is picked is $\frac{4}{10}$, or $\frac{2}{5}$.

➤ Jenny and Morningstar planned an experiment for the class.
Each student would pick a cube from the bag without looking, then replace it.
She would do this 10 times.
Here are the results of one experiment.

Colour	Blue	Red	Green	Yellow
Number of Times	6	1	1	2

The blue cube was picked 6 times.
The **experimental probability** is the likelihood that something occurs based on the results of an experiment.

$$\text{Experimental probability} = \frac{\text{Number of times an outcome occurs}}{\text{Number of times the experiment is conducted}}$$

So, the experimental probability of picking a blue cube is $\frac{6}{10}$, or $\frac{3}{5}$.
This is different from the theoretical probability.

➤ Jenny and Morningstar combined the results from 10 experiments.
Here are the results for 100 trials.

Colour	Blue	Red	Green	Yellow
Number of Times	43	22	18	17

The blue cube was picked 43 times.
So, the experimental probability of picking a blue cube is $\frac{43}{100}$.

The experimental probability is close to the theoretical probability of $\frac{4}{10}$.

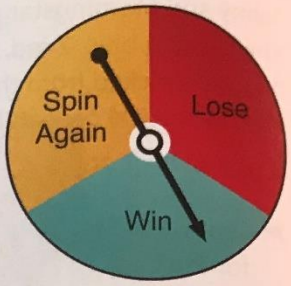
The more trials we conduct, the closer the experimental probability may come to the theoretical probability.

$$\frac{4}{10} \overset{\times 10}{\underset{\times 10}{=}} \frac{40}{100}$$

$\frac{40}{100}$ is close to $\frac{43}{100}$.

Practice

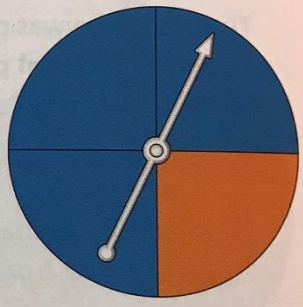
1. For each experiment, state the possible outcomes.
   a) The spinner has 3 equal sectors labelled Win, Lose, Spin Again.
      The pointer on a spinner is spun.
   b) A bag contains 6 marbles: 3 red, 2 black, and 1 blue.
      One marble is picked at random.
   c) A regular tetrahedron has 4 faces labelled 1, 2, 2, 3.
      The tetrahedron is rolled.



2. Dave tossed a coin 20 times. Heads showed 12 times.
   a) How many times did tails show?
   b) What fraction of the tosses showed heads? Tails?
   c) Are these results what you would expect? Explain.
   d) Dave tosses the coin 100 times.
      What would you expect the results to be? Explain.

3. Avril spins the pointer on this spinner several times.
   Here are her results.

Blue	Orange
𝍸 𝍸 𝍸 ||	𝍸 ||



   a) How many times did Avril spin the pointer?
      How do you know?
   b) What fraction of the spins were blue? Orange?
   c) Were Avril's results what you would have expected? Explain.

4. Nina and Allegra placed 35 red tiles and 15 yellow tiles in a bag.
   At random, they picked a tile from the bag, recorded its colour, and replaced it. They did this 100 times.
   a) What is the theoretical probability of picking a red tile?
   b) Predict how many times Nina and Allegra should get a red tile in 100 trials.
   c) Nina and Allegra picked a red tile from the bag 58 times.
      What is the experimental probability of picking a red tile?
   d) Nina said, "I think we did something wrong." Do you agree? Why?
   e) Work with a partner. Try the experiment. Record your results.
      What is your experimental probability of picking a red tile?

5. A die labelled 1 to 6 is rolled.

a) What are the possible outcomes?

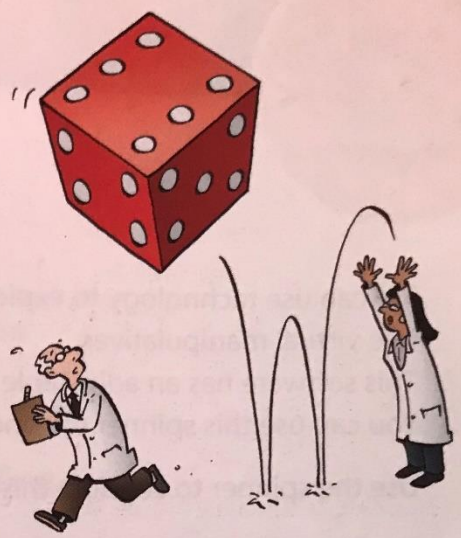
b) What is the theoretical probability of each outcome?

i) rolling a 6

ii) rolling an even number

iii) rolling a 2 or a 4

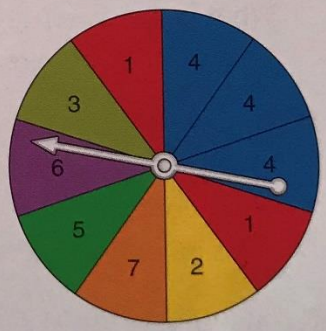
iv) rolling a number greater than 4



c) Work with a partner. Roll a die 20 times. Record your results. What is the experimental probability of each outcome in part b? How do these probabilities compare with the theoretical probabilities? Explain.

d) Combine your results with those of 4 other groups. What is the experimental probability of each outcome in part b? How do these probabilities compare with the theoretical probabilities? Explain. What do you think might happen if you rolled the die 500 times?

6. Zeroun and Ammon are playing a game. They spin the pointer on this spinner. If the pointer lands on an even number, Zeroun wins. If the pointer lands on an odd number, Ammon wins.



a) Is this a fair game? How do you know?

b) What is the theoretical probability of the pointer landing on an even number?

c) Use a spinner like this one. Play the game at least 30 times. Record your results. Were the results what you expected? Explain.

d) What results would you expect if you played the game 100 times? Explain how you made your prediction.

When asked to compare Theoretical and Experimental probabilities, remember it is often easiest to find a common denominator for the two fractions.

For example: P_T (E) = 5/10 P_E (E) = 37/50

*since 10 is a factor of 50, rewrite the theoretical probability as an equivalent fraction out of 50. (multiply both the numerator and denominator by 5).

$$\frac{5_{x5}}{10_{x5}} = \frac{25}{50}$$

We can now see, the Experimental probability was greater than the Theoretical probability for this event.

To follow up with some online activities for Experimental Probabilities, go to www.netmath.ca

a. Comparing the probabilities of events
b. Determining the probability of an event 3
c. Comparing theoretical and experimental probability