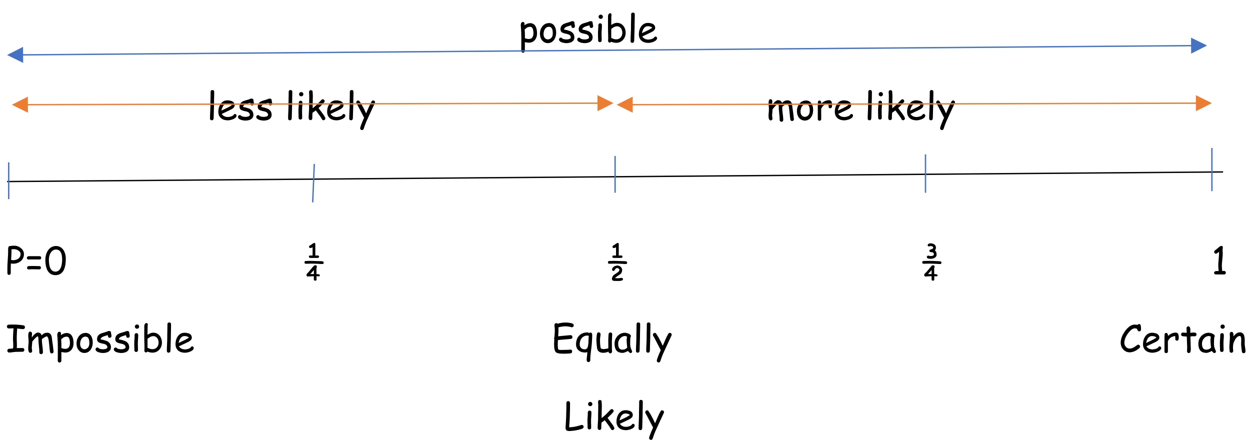
Hello again Grade 8!

This week we will turn our attention to solving Probability problems. To start, lets review some of what we have already learned. Remember:



By definition, Probability is the measure of how likely an event is to occur, It is about predictions of events over the long term.

There are two types of Probability:

a. Theoretical – what should happen. It is represented by

$$P_T(E) = \frac{\text{number of favourable outcomes}}{\text{Number of possible outcomes}}$$

*where (E) represents an event

For example: flip a coin $P_T(\text{Heads}) = \frac{1}{2}$

where 1 represents the chance of getting a head on a coin toss, and 2 is the number of possible outcomes on a coin (heads or tails).

b. Experimental – what actually happens as a result of conducting an experiment. It is represented by

$$P_E(E) = \frac{\text{number of observed occurrences of the favourable outcome}}{\text{sample space (total \# trials of the experiment)}}$$

For example: flip a coin

In theory, we know the probability of getting a head is $\frac{1}{2}$.

If you flip the coin ten times, you do not always get 5H, 5T.

*Remember – the more often we repeat an experiment, the closer we get to theory.

So, if in a given experiment you flip the coin 100 times, and see 36 heads, the probability of getting heads would be represented as

P_E (Heads) = 36/100   *reduce (divide numerator & denominator by 4)

= 9/25

Representing Independent events using Tree Diagrams and Tables

Independent events are defined as two events, such that the outcome of one event has no effect on the outcome of the other.

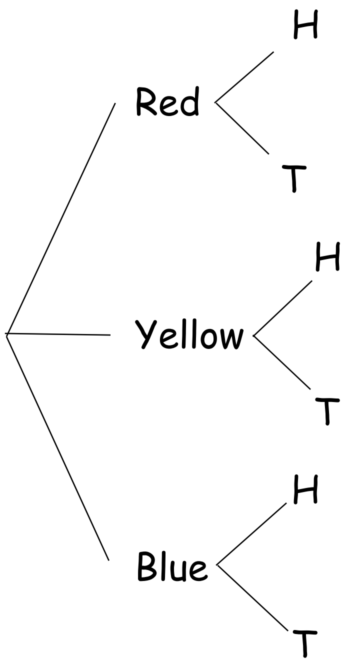
For example: To show the possible outcomes of spinning this spinner and flipping the coin.

This Photo by Unknown

Table:

	RED	YELLOW	BLUE
HEADS	R,H	Y,H	B,H
TAILS	R,T	Y,T	B,T

Tree Diagram:



Sample space: R,H
R,T
Y,H
Y,T
B,H
B,T

Regardless of the model used, there should be the same number of outcomes.

If you were asked to give the probability of getting red on the spinner and heads on the coin, both the table and tree diagram show that there is one chance out of six possible outcomes of getting a R,H.

NOW that we have reviewed our prior knowledge of Probability, we can take the next step. 😊

In the above example, the events are again **independent**, as the result of spinning the spinner will have no effect on the result of flipping the coin (and vice versa).

When this is true, we can also determine the probability of an outcome by considering the probabilities of each independent event.

This is how:

P_T (Red, Heads) = 1/6 we already know this from our table and tree diagram.

Considering the individual probabilities,

P_T (Red) = 1/3 (one red out of three possibilities on the spinner) and
P_T (Heads) = 1/2 (one head out of two possible outcomes on the coin)

Therefore,

$P_T (R,H) = P_T (R) \times P_T (H)$

$= 1/3 \times 1/2$

$= 1/6$

This then, provides us with a **rule** for determining probabilities of independent events:

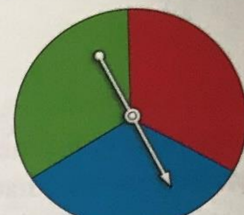
P_T (A and B) = P_T (A) X P_T (B)

I will provide some examples from our text this week with some exercises to follow. Read through the examples and give the circled questions a try. (see next page)

*Remember: Space out the work so you don't try to do too much in one day. Try your best, it's all anyone can ask! 😊

Connect

Two events are **independent events** when one event does not affect the other event.



The pointer on this spinner is spun twice. Landing on red and landing on blue are examples of two independent events.

Use a table to find the probability of landing on red twice.

There are 9 possible outcomes:
RR, RB, RG, BR, BB, BG, GR, GB, GG
Only one outcome is RR.
So, the probability of landing on red twice is $\frac{1}{9}$.

		First Spin		
		R	B	G
Second Spin	R	RR	RB	RG
	B	BR	BB	BG
	G	GR	GB	GG

The probability of landing on red on the first spin is $\frac{1}{3}$.

The probability of landing on red on the second spin is $\frac{1}{3}$.
Note that: $\frac{1}{9} = \frac{1}{3} \times \frac{1}{3}$

probability of landing on red twice = probability of landing on red on the first spin × probability of landing on red on the second spin

This illustrates the rule below for two independent events.

Suppose the probability of event A is written as P(A).
The probability of event B is written as P(B).
Then, the probability that both A and B occur is written as P(A and B).
If A and B are independent events, then: P(A and B) = P(A) × P(B)

Example 1

A coin is tossed and a regular tetrahedron labelled 5, 6, 7, and 8 is rolled.

a) Find the probability of tossing heads and rolling an 8.
b) Find the probability of tossing heads or tails and rolling an even number.

Use a tree diagram to verify your answers.

A Solution

Since the outcome of tossing the coin does not depend on the outcome of rolling the tetrahedron, the events are independent.

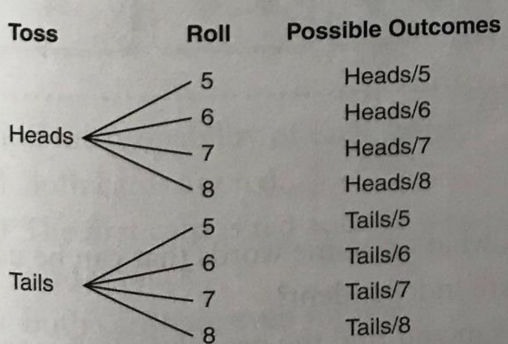
a) When the coin is tossed, there are 2 possible outcomes.
One outcome is heads.
So, P(heads) $= \frac{1}{2}$
When the tetrahedron is rolled, there are 4 possible outcomes.
One outcome is an 8.
So, P(8) $= \frac{1}{4}$

$$\begin{aligned}\text{P(heads and 8)} &= \text{P(heads)} \times \text{P(8)}\\ &= \frac{1}{2} \times \frac{1}{4}\\ &= \frac{1}{8}\end{aligned}$$

b) When the coin is tossed, there are 2 possible outcomes.
Two outcomes are heads or tails.
So, P(heads or tails) $= \frac{2}{2} = 1$
When the tetrahedron is rolled, there are 4 possible outcomes.
Two outcomes are even numbers: 6 and 8
So, P(even number) $= \frac{2}{4} = \frac{1}{2}$

$$\begin{aligned}\text{P(heads or tails and even number)} &= \text{P(heads or tails)} \times \text{P(even number)}\\ &= 1 \times \frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$

Use a tree diagram to check your answers.



There are 8 possible outcomes.
One outcome is heads/8.
So the probability of tossing heads and rolling 8 is $\frac{1}{8}$.
Four outcomes have heads or tails and an even number: heads/6, heads/8, tails/6, tails/8
So, the probability of tossing heads or tails and rolling an even number is $\frac{4}{8}$, or $\frac{1}{2}$.

Example 2

The pocket of a golf bag contains 9 white tees, 7 red tees, and 4 blue tees.
The golfer removes 1 tee from her bag without looking, notes the colour, then returns the tee to the pocket.
The process is repeated.
Find the probability of each event.

a) Both tees are red.

b) The first tee is not red and the second tee is blue.

A Solution

Since the first tee is returned to the pocket, the events are independent.

a) There are 20 tees in the pocket.

$$P(\text{red}) = \frac{7}{20}$$

$$\text{So, } P(\text{red and red}) = P(\text{red}) \times P(\text{red})$$
$$= \frac{7}{20} \times \frac{7}{20}$$
$$= \frac{49}{400}$$

b) $P(\text{not red}) = P(\text{white or blue})$

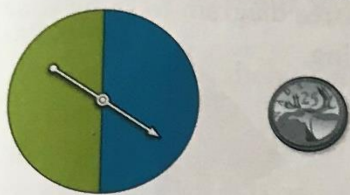
$$= \frac{13}{20}$$

$$\text{So, } P(\text{not red, blue}) = P(\text{not red}) \times P(\text{blue})$$
$$= \frac{13}{20} \times \frac{4}{20}$$
$$= \frac{13}{20} \times \frac{1}{5}$$
$$= \frac{13}{100}$$

Practice

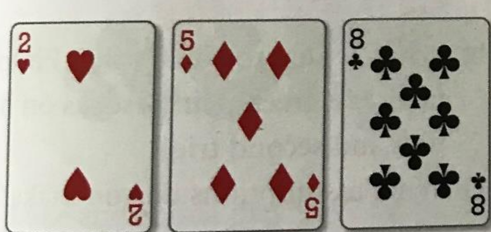
Check

3. A spinner has 2 congruent sectors coloured blue and green. The pointer is spun once, and a coin is tossed.



Find the probability of each event:
a) blue and tails
b) blue or green and heads

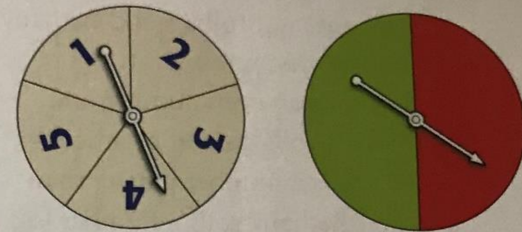
4. Stanley has two sets of three cards face down on a table. Each set contains: the 2 of hearts, the 5 of diamonds, and the 8 of clubs. He randomly turns over one card from each set.



Find the probability of each event:
a) Both cards are red.
b) The first card is red and the second card is black.
c) Both cards are even numbers.
d) The sum of the numbers is greater than 8.

Which strategy did you use each time?

5. Raoul spins the pointer on each spinner. Find the probability of each event.

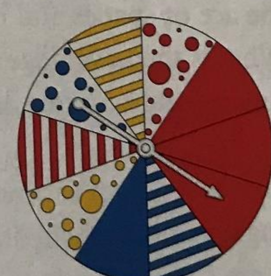


a) green and a 2
b) red and an even number
c) green and a prime number

Use a tree diagram or a table to verify your answers.

Apply

6. Find the probability of each event:
a) i) The pointer lands on a blue spotted sector, then a solid red sector.
ii) The pointer lands on a red sector, then a spotted sector.



iii) The pointer lands on a striped sector, then a solid blue sector.
iv) The pointer lands on a blue or red sector, then a spotted sector.

b) Use a different strategy to verify your answers in part a.

7. Bart and Bethany play a game. They each roll a regular 6-sided die labelled 1 to 6. Find the probability of each event:

a) Each player rolls a 6.

b) Bart rolls a 6 and Bethany rolls a 2.

c) Bart does not roll a 4 and Bethany rolls an even number.

d) Bart rolls an even number and Bethany rolls an odd number.

e) Bart rolls a number greater than 3 and Bethany rolls a number less than 4.

8. An experiment consists of rolling a die labelled 3 to 8 and picking a card at random from a standard deck of playing cards.

a) What is the probability of each event?

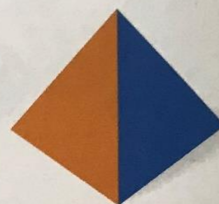
i) rolling a 6 and picking a spade

ii) not rolling a 4 and picking an ace

b) Use a tree diagram to verify your answer to part a, i.

c) What is the probability of picking the ace of spades and rolling a 5? What is the advantage of using the rule instead of a tree diagram?

9. A game at a school carnival involves rolling a regular tetrahedron. Its four faces are coloured red, orange, blue, and green. A player rolls the tetrahedron twice. To win, a player must roll the same colour both times. Marcus has been watching the game. He says he has figured out the probability of a player winning. "The probability of rolling any colour is $\frac{1}{4}$. So, the probability of rolling the same colour again is $\frac{1}{4}$. Since the events are independent, the probability of rolling the same colour both times is $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$."
Do you agree with Marcus?
Justify your answer.
Use a tree diagram to show your thinking.



10. A dresser drawer contains five pairs of socks of these colours: blue, brown, green, white, and black. The socks in each pair are folded together.
Pinto reaches into the drawer and takes a pair of socks without looking.
He wants a black pair.

a) What is the probability that Pinto takes the black pair of socks on his first try?

b) What is the probability that Pinto takes the green pair of socks on his first and second tries?

c) What assumptions do you make?

11. Suppose it is equally likely that a baby be born a boy or a girl.

a) What is the probability that, in a family of 2 children, both children will be boys?

b) Verify your answer to part a using a different method.

Note: The rule for the probability of two independent events can also be used for three or more independent events. Read through the examples on the next pages and try the circled questions that follow. (see next page)

Connect

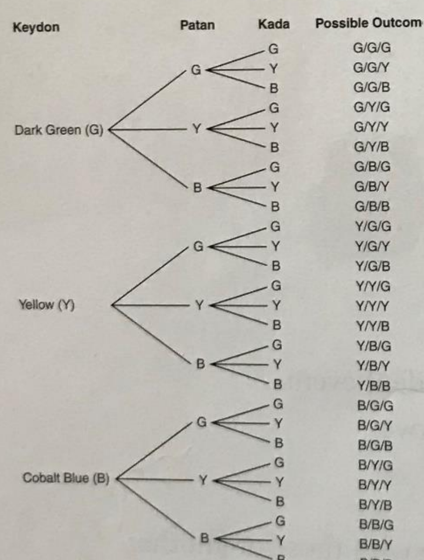
The rule for the probability of two independent events can be extended to three or more independent events.

Example 1

The students in a Grade 8 class were making buffalo horn beaded chokers. Students could choose from dark green, yellow, and cobalt blue Crow beads. Each student has the same number of beads of each colour. Keydon, Patan, and Kada take their first beads without looking. Find the probability that Keydon takes a yellow bead, Patan takes a dark green bead, and Kada takes a yellow bead.

A Solution

Since each student has her own set of beads, the events are independent. Use a tree diagram.



There are 27 possible outcomes. One outcome is Y/G/Y. So the probability that Keydon takes a yellow bead, Patan takes a dark green bead, and Kada takes a yellow bead is $\frac{1}{27}$.

In *Example 1*, the probability that Keydon takes a yellow bead is $P(Y) = \frac{1}{3}$.
The probability that Patan takes a dark green bead is $P(G) = \frac{1}{3}$.
The probability that Kada takes a yellow bead is $P(Y) = \frac{1}{3}$.
Note that: $\frac{1}{27} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$
Suppose the probability of Event A is P(A), the probability of Event B is P(B), and the probability of Event C is P(C).
Then, the probability that all A, B, and C occur is P(A and B and C).
If A, B, and C are independent events,
then P(A and B and C) = P(A) × P(B) × P(C).

Example 2

On a particular day in July, there is a 20% probability of rain in Vancouver, a 65% probability of rain in Calgary, and a 75% probability of rain in Saskatoon.
What is the probability that it will rain in all 3 cities on that day?

A Solution

The events are independent.
Write each percent as a decimal, then multiply the decimals.
P(Rain in Vancouver) = 20%, or 0.20
P(Rain in Calgary) = 65%, or 0.65
P(Rain in Saskatoon) = 75%, or 0.75
So, P(Rain V and C and S) = P(Rain V) × P(Rain C) × P(Rain S)
= 0.20 × 0.65 × 0.75
= 0.0975, or 9.75%

Use a calculator.

The probability that it will rain in all 3 cities on that day is 9.75%.

Example 2
Another Solution

Assume the events are independent.
P(Rain in Vancouver) = 20%, or $\frac{20}{100} = \frac{1}{5}$
P(Rain in Calgary) = 65%, or $\frac{65}{100} = \frac{13}{20}$
P(Rain in Saskatoon) = 75%, or $\frac{75}{100} = \frac{3}{4}$
So, P(Rain V and C and S) = P(Rain V) × P(Rain C) × P(Rain S)
$= \frac{1}{5} \times \frac{13}{20} \times \frac{3}{4}$
$= \frac{39}{400}$

The probability that it will rain in all 3 cities on that day is $\frac{39}{400}$.

Note that $\frac{39}{400} = 0.0975$.

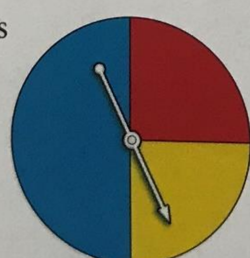
Practice

Check

4. One coin is tossed 3 times. Find the probability of each event:
   a) 3 heads
   b) 3 tails
   c) tails, then heads, then tails
   Use a tree diagram to verify your answers.

5. A red die, a blue die, and a green die are rolled. Each die is labelled 1 to 6. Find the probability of each event:
   a) a 2 on the red die, a 3 on the blue die, and a 4 on the green die
   b) a 4 on the red die, an even number on the blue die, and a number less than 3 on the green die

6. A spinner has 3 sectors coloured red, blue, and yellow. The pointer on the spinner is spun 3 times. Find the probability of each event:



   a) red, blue, and yellow
   b) blue, blue, and not red
   c) blue, blue, and blue

Apply

7. Stanley's bicycle lock has 4 dials, each with digits from 0 to 9. What is the probability that someone could guess his combination on the first try by randomly selecting a number from 0 to 9 four times?

8. A coffee shop has a contest. When you "lift the lid," you might win a prize. The probability of winning a prize is $\frac{1}{10}$. Suppose your teacher buys one coffee each day. Find the probability of each event:
   a) Your teacher will win a prize on each of the first 3 days of the contest.
   b) Your teacher will win a prize on the third day of the contest.
   c) Your teacher will not win a prize in the first 4 days of the contest.

9. **Assessment Focus** Nadine, Joshua, and Shirley each have a standard deck of playing cards. Each student randomly draws a card from the deck. Find the probability of each event:

a) Each student draws a heart.

b) Nadine draws a spade, Joshua draws a spade, and Shirley draws a red card.

c) Nadine does not draw a heart, Joshua draws a black card, and Shirley draws an ace. Show your work.

10. Preet writes a multiple-choice test.
The test has 5 questions.
Each question has 4 possible answers.
Preet guesses each answer.
Find the probability of each event:

a) She answers all 5 questions correctly.

b) She answers only the first 3 questions correctly.

c) She answers all the questions incorrectly.

11. Rocco chooses a 3-letter password for his e-mail account. He can use a letter more than once. What is the probability that someone else can access his e-mail by randomly choosing 3 letters?

12. Vanessa has 16 songs on a Classic Rock CD. Six of the songs are by the Beatles, 4 are by the Rolling Stones, 4 are by the Who, and 2 are by the Doors. Vanessa plays the CD. She selects a setting that randomly chooses songs to play.
Find the probability of each event:

a) The first 3 songs played are by the Beatles.

b) The first 2 songs played are by the Rolling Stones and the next 2 songs are by the Beatles.

c) The first 2 songs played are by the Doors, and the next song played is either by the Beatles or the Rolling Stones.

13. A bag contains 5 blue marbles and 1 white marble. Susan draws a marble from the bag without looking, then replaces it in the bag. This is done 5 times.

a) What is the probability that the white marble is drawn 5 times in a row? Express your answer as a percent.

b) Suppose the white marble is drawn 5 times in a row. What is the probability the white marble will be picked on the next draw? Explain.

c) Is your answer to part b the same as the probability of drawing the white marble 6 times in a row? Why or why not?

Have a fantastic week! 😊