Hello everyone!

This week we will finish with a lesson on the area of triangles and parallelograms, followed by a look at transformations.

We started our work with area last week, finding the area of circles.

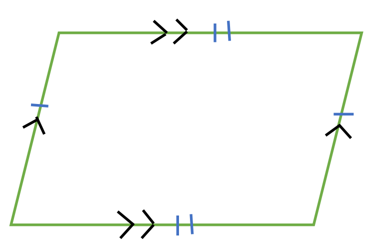
To review:

Area is the measure of the number of square units to cover a region. (measured in square units, mm^2, cm^2, etc.)

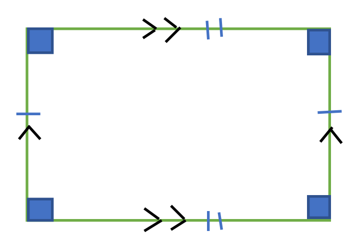
To find the area of a parallelogram, let's first understand the relationship between a rectangle and parallelogram. We know already, from previous years that $A = bh$ (A = base x height) for a rectangle.

We also know that a rectangle is a special kind of parallelogram. Let's look:

Parallelogram

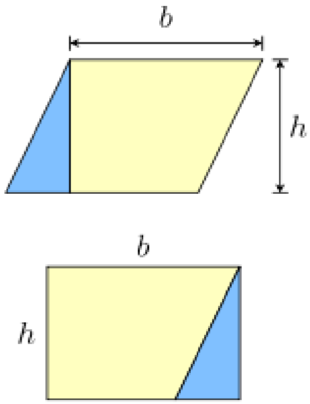


Rectangle



Both shapes have 2 pairs of equal, parallel sides and opposite angles are equal. The difference in the rectangle, is that all angles are 90°, as the base is perpendicular to the height.

Imagine cutting one corner off the parallelogram, and putting it on the other side. What do you think it would make? Take a look on the following page and see

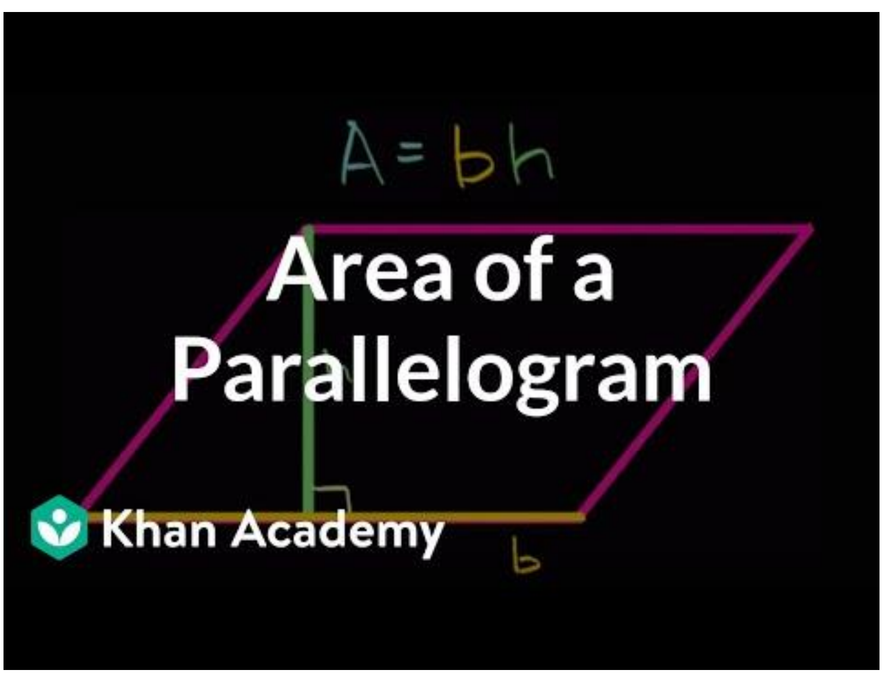


Note: the height of a parallelogram is found by measuring the length of the line that joins two of the parallel sides and is perpendicular to the base.

It is easy to see that the blue corner removed on the left side of the parallelogram, creates a rectangle when placed on the right side. This shows us then, that the area of the surface covered by the parallelogram will be the same as for a rectangle.

The area of a parallelogram then, is also: $A = bh$, where the height is perpendicular to the base

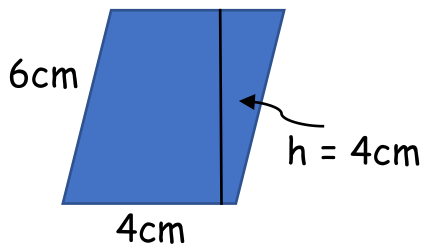
To review, watch the following video:



We can also find the length of a missing dimension, if given the area and either the base or the height.

Since $A = b \times h$, then $b = A \div h$ and $h = A \div b$

Let's try an example:



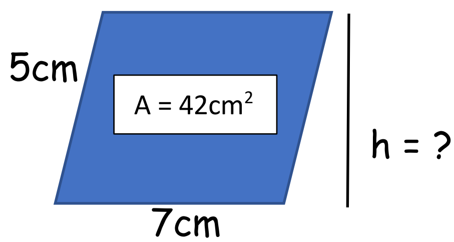
$A = bh$

$= 4cm \times 4cm$

$= 16cm^2$

* Be careful, the height is <u>perpendicular to the base</u>.

or



$A = bh$

$42cm^2 = 7cm \times h$

$h = 42cm^2 \div 7cm$

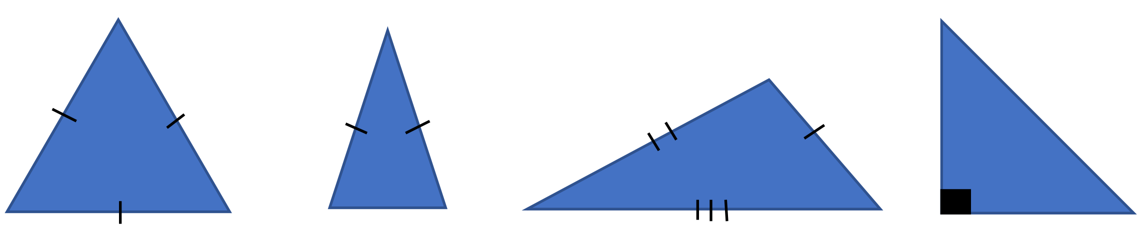
$= 6cm$

*the side length is NOT the height!

Take some time to try some on your own on Netmath at www.netmath.ca The activity is "Calculating the area of parallelograms and rhombuses 1"

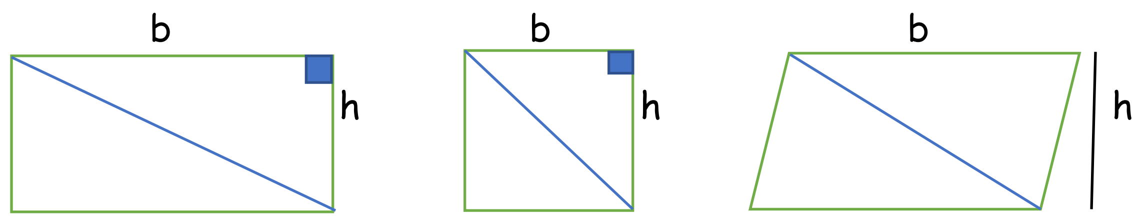
To find the area of a triangle, let's first review some background information.

A triangle is a 3 sided polygon, including:



Equilateral Isosceles Scalene Right

To find the area of a triangle, let's first understand the relationship between a triangle and a rectangle, square, or parallelogram.



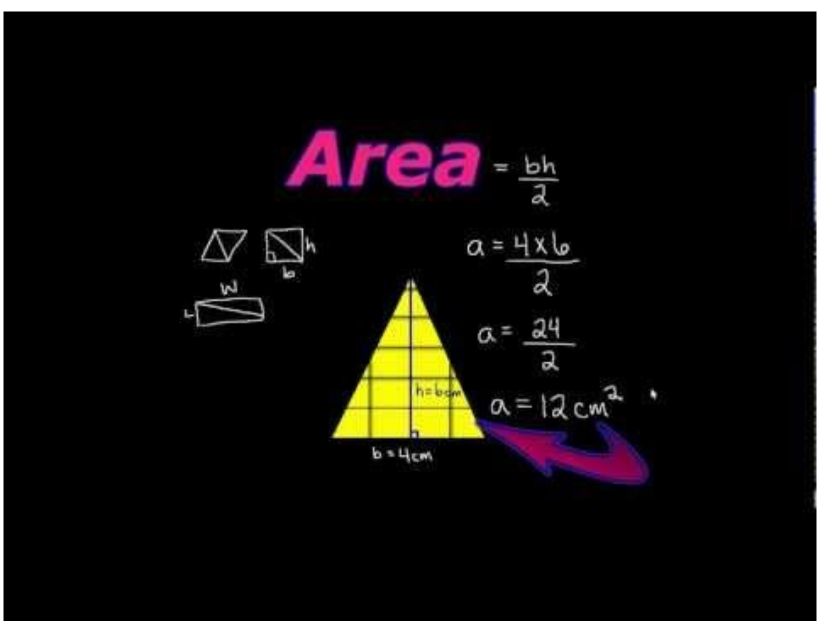
Notice when you draw a diagonal in either of the shapes, it creates two <u>congruent</u> (equal) triangles. If the triangles are congruent, they also have the same area.

As a result, we know the area of both triangles is the area of the rectangle, square or parallelogram, $A = bh$

To find the area of just one triangle, take half the area of the rectangle, square or parallelogram, so the area of a triangle is found by:

$A = \frac{b \times h}{2}$ *where the height is ALWAYS perpendicular to the base

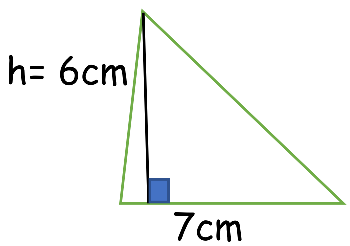
Again, listen to the following video to be clear:



We can also find the length of a missing dimension, if given the area and either the base or the height.

For triangles though, if you double the area of the given triangle first (make the rectangle again), you can easily find the missing dimension using your area model for a rectangle.

Let's try some examples:



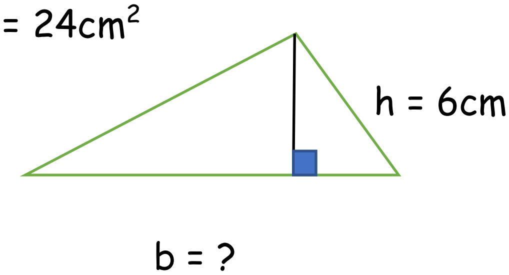
$$A = \frac{bh}{2}$$

$$= \frac{7cm \times 6cm}{2}$$

$$= \frac{42cm^2}{2}$$

$$= 21cm^2$$

$A = 24cm^2$



*to solve for the base, remember the area of the triangle is <u>half</u> of the parallelogram it would make if we put another triangle on top.

The area of the parallelogram would

$24cm^2 \times 2 = 48cm^2$

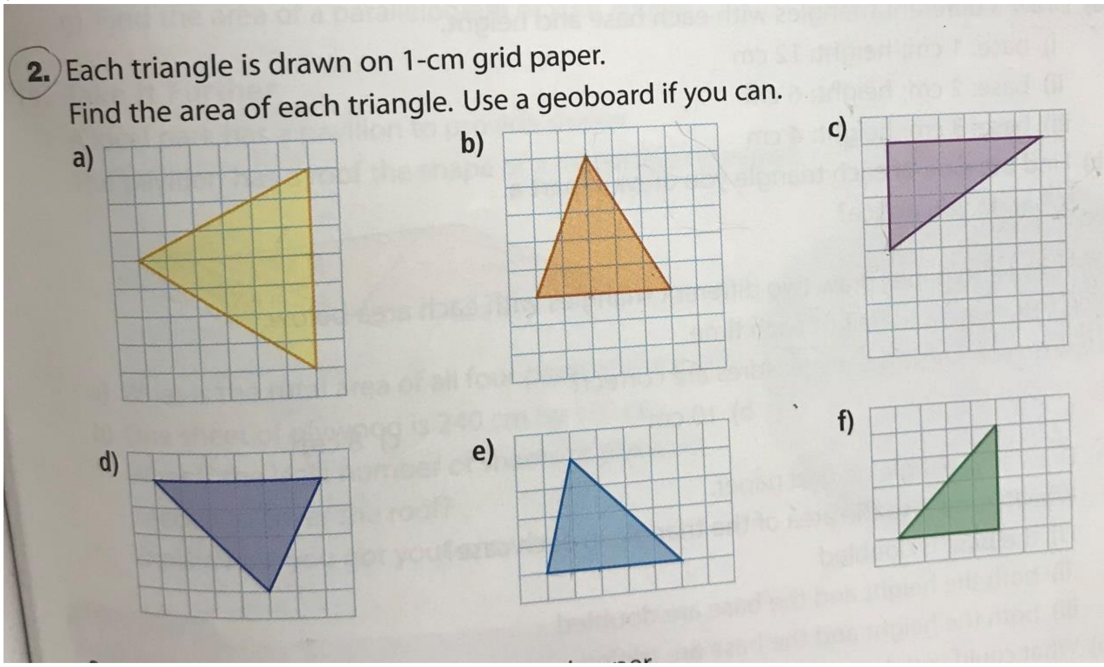
Using A = bh, we can now find the height of the triangle

$48cm^2 = b \times 6cm$

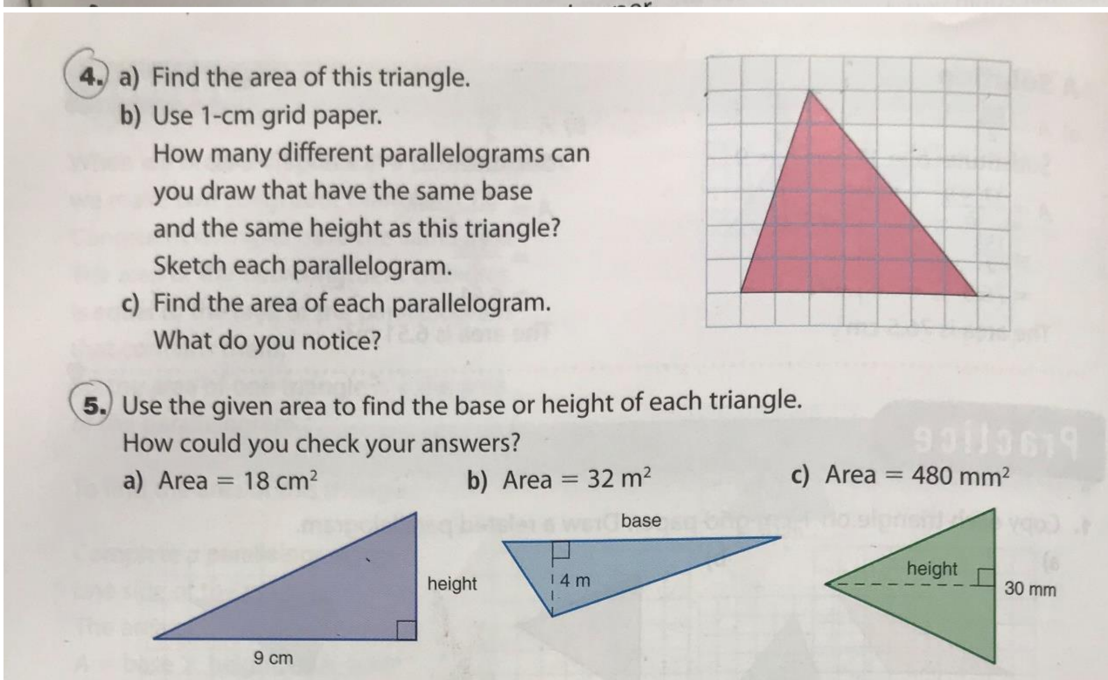
$b = 48cm^2 \div 6cm$

$= 8cm$

Let’s try some on our own from the text this time. You do not need to do every a,b,c in each question when they are the same type of problem.



2. Each triangle is drawn on 1-cm grid paper.
Find the area of each triangle. Use a geoboard if you can.



4. a) Find the area of this triangle.
b) Use 1-cm grid paper.
How many different parallelograms can you draw that have the same base and the same height as this triangle?
Sketch each parallelogram.
c) Find the area of each parallelogram.
What do you notice?

5. Use the given area to find the base or height of each triangle.
How could you check your answers?
a) Area = 18 cm^2
b) Area = 32 m^2
c) Area = 480 mm^2

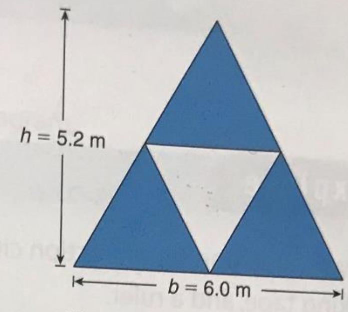
(see next page)

9. Assessment Focus

This triangle is made from 4 congruent triangles.
Three triangles are to be painted blue.
The fourth triangle is not to be painted.

a) What is the area that is to be painted?
Show your work.

b) The paint is sold in 1-L cans.
One litre of paint covers 5.5 m^2.
How many cans of paint are needed?
What assumptions did you make?



The height is approximate.

10. Look at the diagram to the right.

a) How many triangles do you see?

b) How are the triangles related?

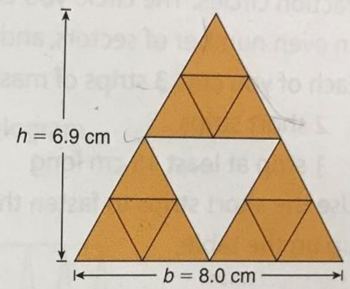
c) How many parallelograms do you see?

d) Find the area of the large triangle.

e) Find the area of one medium-sized triangle.

f) Find the area of one small triangle.

g) Find the area of a parallelogram of your choice.



The height is approximate.

The last part of this week's lesson is on Transformations.

There are 3 transformations: Translations (slides)

Reflections (flips)

Rotations (turns)

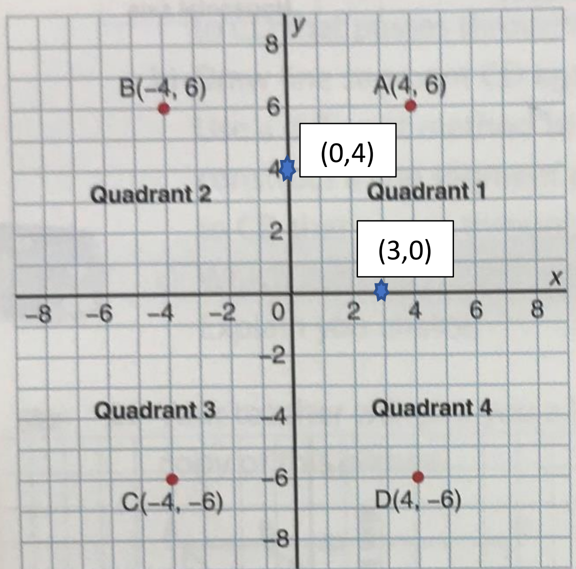
Let's review first what we know about the Cartesian plane, also called the Co-ordinate grid as we will be plotting points again.

Read through the notes on the next page to refresh your memory.

Read well, we have done this before! 😊

A vertical number line and a horizontal number line that intersect at right angles at 0 form a **coordinate grid**.
The horizontal axis is the ***x*-axis**.
The vertical axis is the ***y*-axis**.
The axes meet at the **origin**, (0, 0).
The axes divide the plane into four **quadrants**.
They are numbered counterclockwise.

This coordinate grid is also called a *Cartesian plane*.



We do not need arrows on the axes.

A pair of coordinates is called an *ordered pair*.

*Don't forget to be careful when plotting points that have a zero co-ordinate.

Ex: To plot (0,4), start at zero (0) on the x-axis, then move up 4 units on the y-axis.

To plot (3,0), start at 3 on the x-axis, then stay there as the y-axis value is zero (0).

In Quadrant 1, to plot point A, start at 4 on the *x*-axis and move up 6 units.
Point A has coordinates (4, 6).
In Quadrant 2, to plot point B, start at −4 on the *x*-axis and move up 6 units.
Point B has coordinates (−4, 6).
In Quadrant 3, to plot point C, start at −4 on the *x*-axis and move down 6 units.
Point C has coordinates (−4, −6).
In Quadrant 4, to plot point D, start at 4 on the *x*-axis and move down 6 units.
Point D has coordinates (4, −6).

We do not have to include a + sign for a positive coordinate.

*Remember, when plotting points, we always plot x-values first, then the y-value.

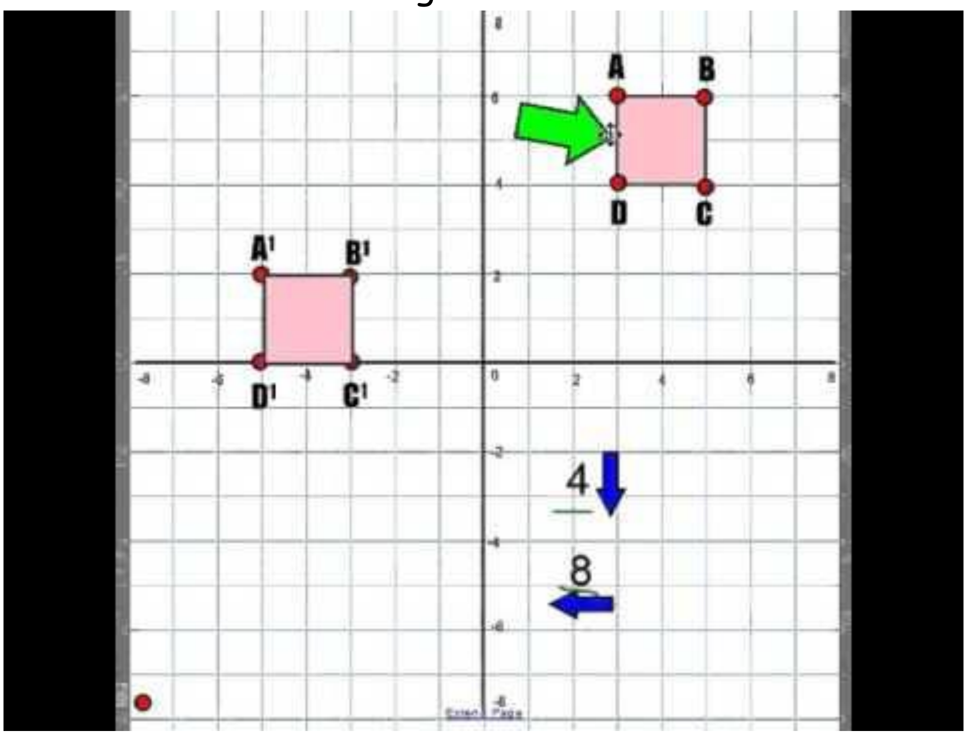
To begin, we will look at Translations (slides):

A translation is a transformation that moves a point or a shape (series of points), in a straight line to another position on the same surface.

For a shape, move each vertex of the shape by the same translation, then reconnect the points.

A translation requires direction; up, down, left or right. For example, translate point A, 2R 1D (which means 2 to the right, then 1 down). The new image A is identified as $A^{|}$ image, and we must be able to identify the co-ordinates of the new image point.

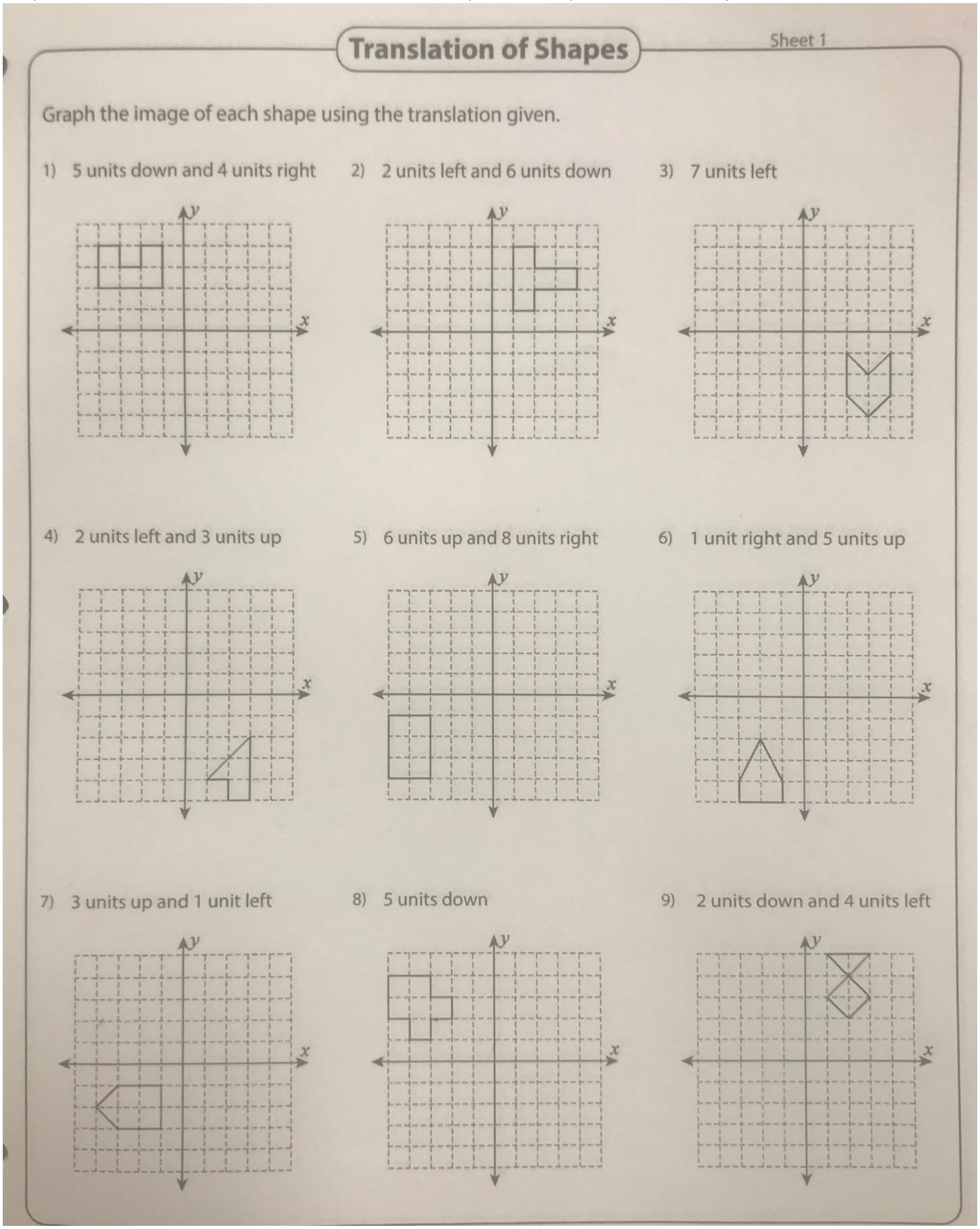
Let's watch the following video to see the translation of a shape:



*Remember, the square was an easy shape to translate. If the shape is more complex, as mentioned in the video, translate each vertex (label as you go), and join the points to reform the new image.

To practice translations, go to Netmath at www.netmath.ca and try the exercise "Performing translations on a Cartesian plane 2"

If you would like to try on paper, and have a printer, you may prefer to try: (label each vertex on the shapes, so you can keep track!)



Translation of Shapes

Sheet 1

Graph the image of each shape using the translation given.

1) 5 units down and 4 units right

2) 2 units left and 6 units down

3) 7 units left

4) 2 units left and 3 units up

5) 6 units up and 8 units right

6) 1 unit right and 5 units up

7) 3 units up and 1 unit left

8) 5 units down

9) 2 units down and 4 units left

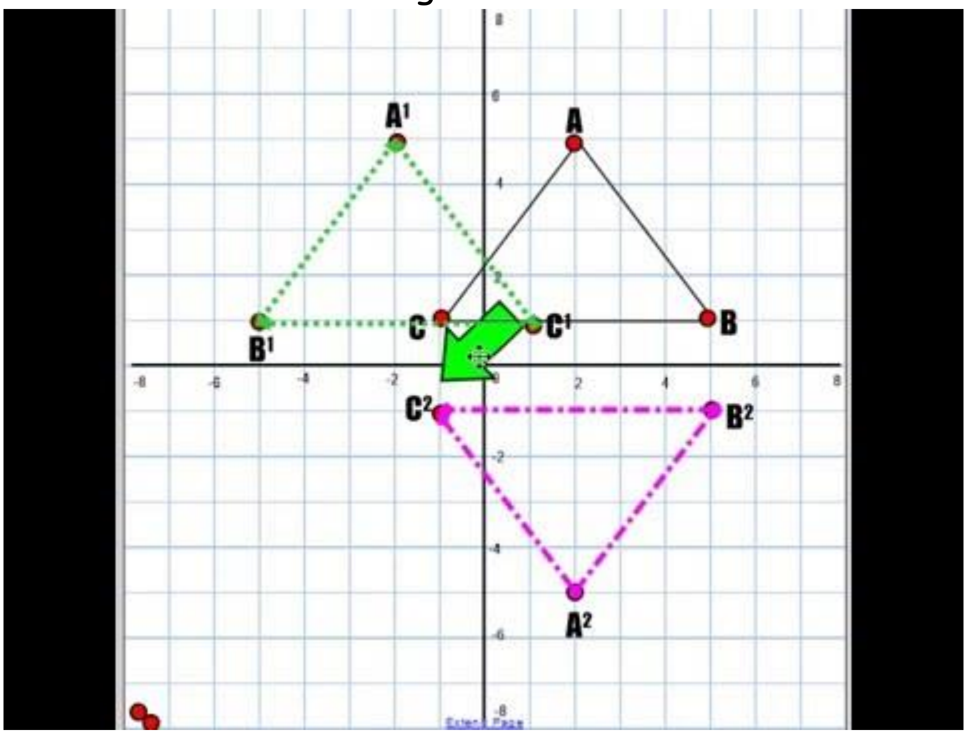
Let's look at Reflections (flips):

A reflection is a transformation that is illustrated by moving a shape across a mirror line to form an image.

The mirror line can be a horizontal, vertical, or even a diagonal line. Often, shapes are reflected across the x-axis or the y-axis.

Every point travels the same distance across the mirror line at 90°.

Let's watch the following video to see the reflection of a shape:



Remember:

When reflecting across a horizontal line or x-axis, each point moves the same distance above or below the mirror line.

When reflecting across a vertical line or y-axis, each point moves the same distance to the right or left of the mirror line.

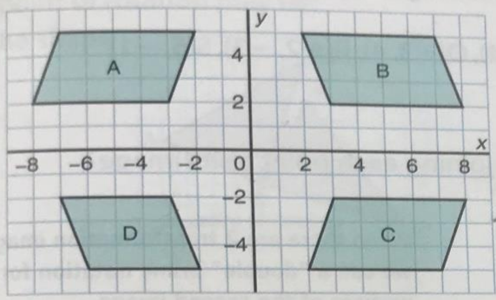
To practice reflections, go to Netmath at www.netmath.ca and try the exercise "Performing reflections on a Cartesian plane 1"

If you would like to try on paper, and have a printer, you may prefer to try: (label each vertex on the shapes, so you can keep track!)

2. Describe the horizontal and vertical distance required to move each point to its image.

a) A(5, −3) to A′(2, 6) b) B(−3, 0) to B′(−5, −3) c) C(2, −1) to C′(4, 3)
d) D(−1, 2) to D′(−4, 0) e) E(3, 3) to E′(−3, 3) f) F(4, −2) to F′(4, 2)

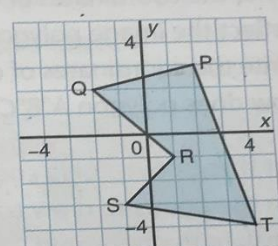
3. The diagram shows 4 parallelograms.



a) Are any 2 parallelograms related by a translation? If so, describe the translation.
b) Are any 2 parallelograms related by a reflection? If so, describe the reflection.

4. Copy this pentagon on grid paper.
Write the coordinates of each vertex.
After each transformation:

- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the pentagon.



a) Draw the image after a translation 3 units left and 2 units up.
b) Draw the image after a reflection in the x-axis.
c) Draw the image after a reflection in the y-axis.

*You will need grid paper

q. 5 Make the co-ordinate grid, then plot the points

5. Plot these points on a coordinate grid:
A(1, 3), B(3, −2), C(−2, 5), D(−1, −4), E(0, −3), F(−2, 0)

a) Reflect each point in the x-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

b) Reflect each point in the y-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

c) How could you use the patterns in parts a and b to check that you have drawn the reflection image of a shape correctly?

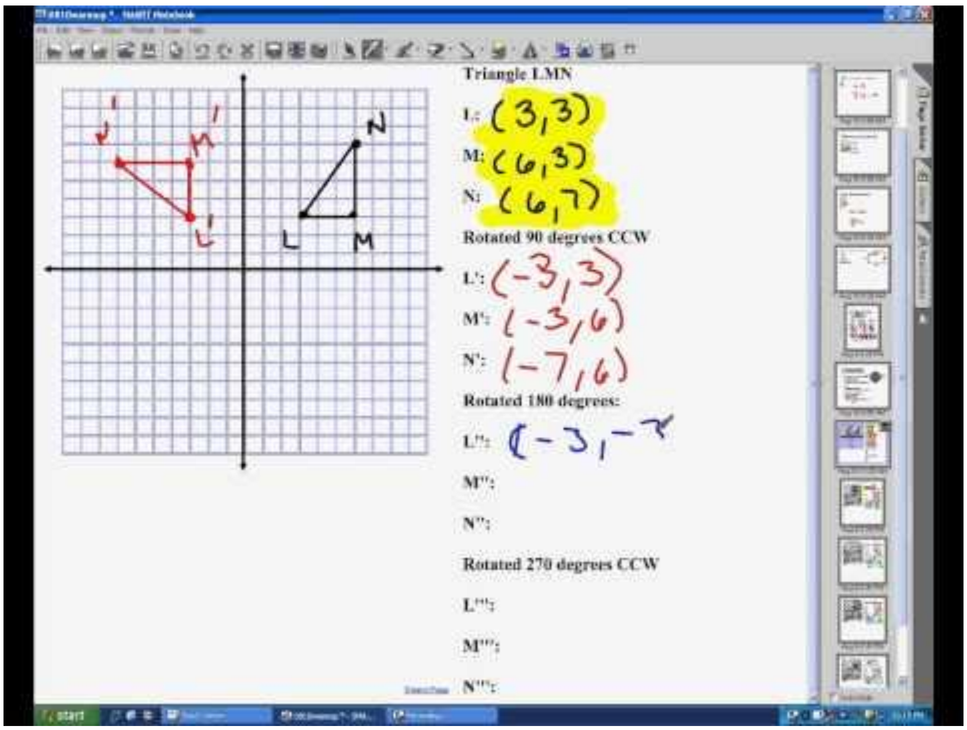
Lastly, let's look at Rotations (turns):

A rotation is a transformation in which a shape is turned about a fixed point.

A rotation turns the shape about a point of rotation that may be either on the shape, or about a point not on the shape.

The rotation may be clockwise (CW) or counterclockwise (CCW).

Let's watch the following video to see the rotation of a shape:



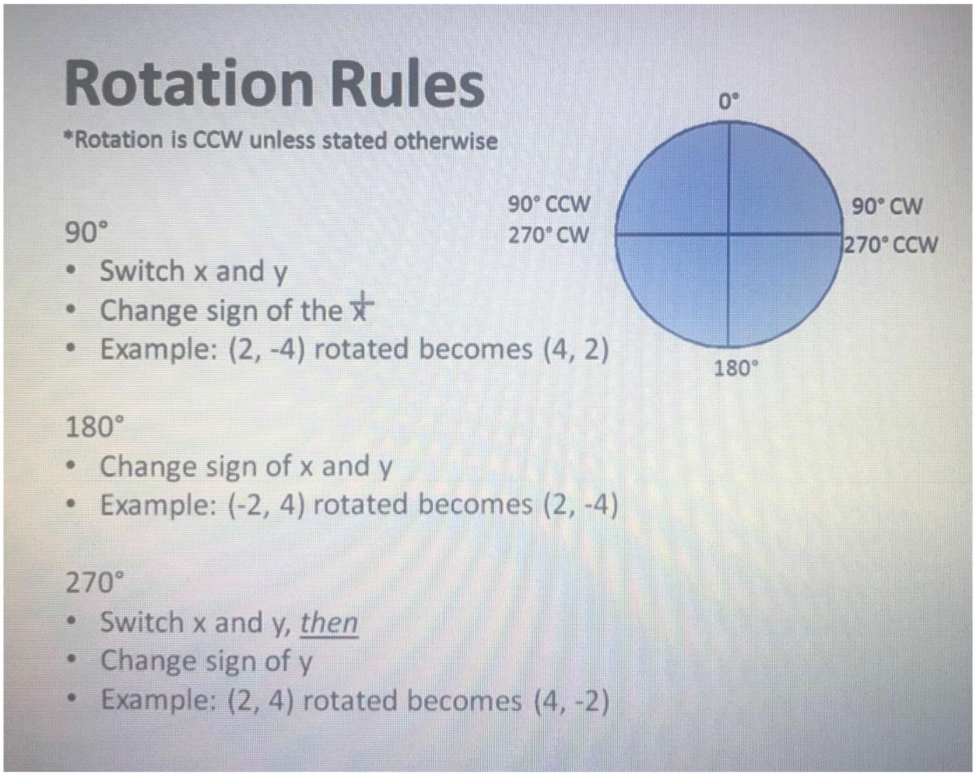
To review: Rotations are done here, by following what was called the "rotation rules". I will add a picture of the rules for you to use when trying your own.

*Remember: 90°CCW is the same as a 270° CW turn

180° is the same either way

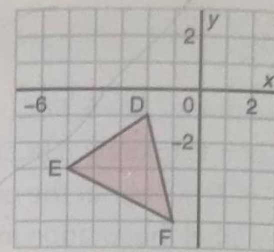
270°CCW is the same as a 90° CW turn

A CCW rotation is shown as +90°, while a CW rotation is shown as -90°



To practice rotations, try the questions on the following page. You will need graph paper.

3. a) Copy $\triangle DEF$ on grid paper.
Write the coordinates of each vertex.
After each rotation:
- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the triangle.



b) Rotate $\triangle DEF$ $-90°$ about the origin to its image $\triangle D'E'F'$.
c) Rotate $\triangle DEF$ $+270°$ about the origin to its image $\triangle D''E''F''$.
d) What do you notice about the images in parts b and c?
Do you think you would get a similar result with any shape that you rotate $-90°$ and $+270°$? Explain.

4. Plot each point on a coordinate grid:
A(2, 5), B(−3, 4), C(4, −1)
a) Rotate each point 180° about the origin O to get image points A′, B′, C′.
Write the coordinates of each image point.
b) Draw and measure:
i) OA and OA′ ii) OB and OB′ iii) OC and OC′
What do you notice?
c) Measure each angle.
i) ∠AOA′ ii) ∠BOB′ iii) ∠COC′
What do you notice?
d) Describe another rotation of A, B, and C that would result in the image points A′, B′, C′.

5. Repeat question 4 for a rotation of −90° about the origin.

6. **Assessment Focus**
a) Plot these points on a coordinate grid:
A(6, 0), B(6, 2), C(5, 3), D(4, 2), E(2, 2), F(2, 0)
Join the points to draw polygon ABCDEF.
b) Translate the polygon 6 units left and 2 units up.
Write the coordinates of each vertex of the image polygon A′B′C′D′E′F′.
c) Rotate the image polygon A′B′C′D′E′F′ 90° counterclockwise about the origin.
Write the coordinates of each vertex of the image polygon A″B″C″D″E″F″.
d) How does polygon A″B″C″D″E″F″ compare with polygon ABCDEF?

Finally, on Dreambox at, https://play.dreambox.com/

there is a last activity on all the transformations. Remember to look for the blue calendar.

This is your last lesson for the year! As always, take your time, space out the work and do your best. What we haven't covered, we will pick up in the fall. Have a fantastic week, and an amazing summer! 😊